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1991 J. Phys.: Condens. Matter 3 9569

(http://iopscience.iop.org/0953-8984/3/47/030)

J. Phys.: Condens. Matter 3 (1991) 9569–9573. Printed in the UK

LETTER TO THE EDITOR

Dynamics of interacting flux kinks in layered high-T_c superconductors

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Received 5 August 1991

Abstract. We have studied the dynamical behaviour of flux kinks, which appear in layered high-T_c superconductors when the magnetic field is parallel and slightly inclined to the plane of the layers. We have obtained the equation of motion for flux kink position from the equation of motion for the displacement field of the flux line. Having performed computer simulations of the resulting equation, we discuss the time-dependent behaviour of the system with the initially random distributed kinks.

It is widely established that the dynamics of the magnetic flux play an important role in the resistive behaviour of high-temperature superconductors (HTSCs) in a magnetic field. Recently Ivlev and Kopnin have pointed out that there is an intrinsic pinning mechanism in layered HTSCs for the flux motion in the direction perpendicular to the layers [1, 2]. Such pinning originates from magnetic flux interactions with the layered structure. For the case with the applied magnetic field parallel and slightly inclined to the plane of layers, they have also predicted the existence of the flux kink where the flux line passes from one valley of the potential (defined below in equation (4)) to another, and have then studied the effect of a single moving flux kink on the current–voltage (I–V) curve [2]. The aim of the present letter is to discuss the effect of interaction among flux kinks on the resistivity, which was neglected by Ivlev and Kopnin.

We consider a system such that the z axis is taken to be along the crystal c axis, the magnetic field is directed along the y axis, H_y, and the transport current, j, flows along the x axis. In addition to H_y, a small magnetic field, H_z, is applied along the z axis. In this situation, a vortex line will be inclined from the y axis; this can then be described by the displacement u_z. Assuming that u_z is a function of y and time, t, only, Ivlev and Kopnin have derived the equation of motion for u_z [2]:

$$\eta \frac{\partial u_z}{\partial t} - C_{44} \frac{\partial^2 u_z}{\partial y^2} - \frac{H_y j}{c} + \frac{H_y j_c}{c} \sin\left(2\pi \frac{u_z}{s}\right) = 0 \tag{1}$$

where η is the kinetic coefficient, C_{44} is the elastic modulus of the flux line lattice, j_c is the critical current, and s is the interlayer distance. Here the third term in (1) denotes the Lorentz force and the last term denotes the pinning force due to the layers. Without loss of generality we have here set the quasimomentum $p = 0$, defined in equation (20) of [2]. (For detailed notation, see [2].) Note that the above equation can be rewritten in the form

$$\eta \frac{\partial u_z}{\partial t} = -\frac{\delta F[u_z]}{\delta u_z} \tag{2}$$

where the free energy of the system, $F[u_z]$, is defined by

$$F[u_z] = \int \mathrm{d}y \left[\frac{C_{44}}{2} \left(\frac{\partial u_z}{\partial y} \right)^2 + V(u_z) \right] \tag{3}$$

with the potential $V(u_z)$

$$V(u_z) = -(H_y j/c)u_z + (sH_y j_c/2\pi c)\left[1 - \cos\left(2\pi \frac{u_z}{s}\right)\right]. \tag{4}$$

A moving one-kink solution for u_z is obtained from (1) by making the *ansatz*

$$u_z(y,t) \equiv M(y - vt). \tag{5}$$

Hereafter we will often use the word 'kink' as a generic term, including both kink and antikink. The function $M(y)$ satisfies the following equation with $V' \equiv \delta V/\delta u_z$:

$$-\eta v \frac{\mathrm{d}M}{\mathrm{d}y} = C_{44} \frac{\mathrm{d}^2 M}{\mathrm{d}y^2} - V'(M) \tag{6}$$

with the boundary conditions

$$M(y) \rightarrow \begin{cases} u^\alpha & \text{if } y \rightarrow -\infty \\ u^\beta & \text{if } y \rightarrow +\infty \end{cases} \tag{7}$$

where u^α and u^β are some of the stationary points, satisfying $V'(u^\alpha) = V'(u^\beta) = 0$. The kink solutions, having the lowest excitation energy with $|u^\alpha - u^\beta| = s$, are known to be stable if u^α and u^β take one of the values, $u^0 + ns$, with n being an integer and $0 < u^0 \equiv (s/2\pi)\sin^{-1}(j/j_c) < s/4$. One-kink velocity v in (5) is determined as an eigenvalue of (6). Based on such a moving one-kink solution of u_z for a weak-current limit and neglecting kink–kink interactions, Ivlev and Kopnin have calculated the induced electric field due to the kink motion [2].

In the following we discuss the time-dependent behaviour of interacting kinks from the initial random distributed kinks, and study such effects on the induced electric field. For this purpose we first obtained the equation of motion for kink positions, numbered from left to right, from the original equation (1) by using the reductive perturbation method [3]. To do so we must assume that: (i) the kink positions are well defined so that kink width (defined below in (12)) is very narrow; (ii) the distance between the neighbouring kinks is large so that we can use the binary interaction approximation; (iii) the transport current is small such that kinks move slowly in time; and (iv) no creation of kinks occurs due to the small thermal energy compared with the nucleation energy. The reductive perturbation method is based on the following approximation for u_z: near the ith kink position, denoted as $y_i(t)$, the profile of u_z is approximated by a superposition of independent kinks as

$$u_z \simeq M_i(y - y_i) + \sum_{j>i} [M_j(y - y_j) - M_j(-\infty)] + \sum_{j<i} [M_j(y - y_j) - M_j(+\infty)]. \tag{8}$$

Here the one-kink solution of (6), located at $y_i(t)$ with velocity v_i, is denoted as M_i. A similar technique has been used for various models [4–7]. So here we quote only the final result—a detailed discussion about deriving the following equation is presented elsewhere. The equation of motion for $y_i(t)$ is given by

$$\gamma\left(\frac{\mathrm{d}y_i}{\mathrm{d}t} - v_i\right) = R(i-1, i) - R(i, i+1) \tag{9}$$

with

$$R(n, m) \equiv (\pi/w)\varepsilon_n \varepsilon_m \exp[-(y_m + v_m t - y_n - v_n t)/w] \tag{10}$$

$$\gamma \equiv \eta/C_{44} \tag{11}$$

$$w \equiv 1/\sqrt{g} \tag{12}$$

$$g \equiv 2\pi H_y j_c/csC_{44} \tag{13}$$

where $\varepsilon_i = 1$ and -1 correspond to a kink and an antikink, respectively, and w denotes the kink width. To obtain the above equation we have used the approximated solution of (6), valid only for the weak-j limit;

$$v_i \simeq \varepsilon_i(\pi/4)(\sqrt{g}/\gamma)(j/j_c) \equiv \varepsilon_i v^0 \tag{14}$$

$$M_i(y) = (s/2\pi)4\varepsilon_i \tan^{-1} \exp[-y/w]. \tag{15}$$

From the above result we can see that the kink–kink interaction is repulsive (attractive) for the same-(different-) type kink pair. Note that several approximations used above require that

$$v^0 \ll w/\tau \equiv V \tag{16}$$

$$w \ll s \tag{17}$$

with

$$\tau \equiv \gamma/\pi g. \tag{18}$$

Note also that in the present situation the induced electric field along the x axis, E_x, is given by

$$E_x = (B_z/c)v^0 A(t) \tag{19}$$

with a normalized electric field, $A(t)$, defined by

$$A(t) \equiv \frac{1}{v^0} \sum_{i=1}^{N(t)} \left|\frac{\mathrm{d}y_i}{\mathrm{d}t}\right| \tag{20}$$

$$N(t) \equiv N_{\mathrm{K}}(t) + N_{\mathrm{AK}}(t) \tag{21}$$

where B_z ($\ll H_y$) is the induced magnetic field due to the distortion of the flux line lattice [2] and is assumed for simplicity to have the same value for each kink, and $N(t)$ denotes the total number of kinks at time t with the flux kink number $N_{\mathrm{K}}(t)$ and the antikink number $N_{\mathrm{AK}}(t)$.

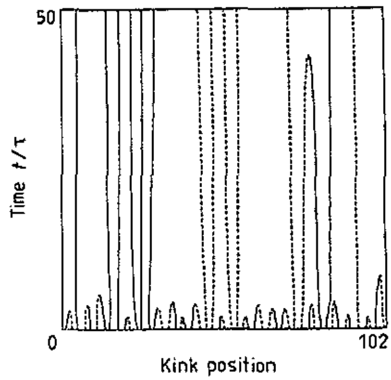

Figure 1. Time variation of kink positions for the initial kink number $N_K(0) = 26$, the initial antikink number $N_{AK}(0) = 25$, and the initial number density of kinks $n_0 = 0.5$. Solid lines denote the trajectories of kinks and dashed lines denote the trajectories of antikinks.

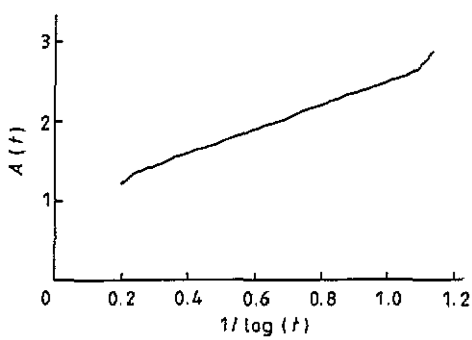

Figure 2. Time variation of the normalized electric field, $A(t)$, defined in equation (20) against $1/\log_{10}(t)$. The unit of time is taken to be τ, defined in equation (18).

The aforementioned kink equation of motion is still highly non-linear and cannot be solved analytically, but is suitable for computer simulations, compared with the original equation (1). We will now solve the above equation of motion numerically by using the Runge–Kutta–Gill method. In the following, the unit of length is taken to be the kink width, w, and the unit of time is τ. We assume that kinks and antikinks annihilate each other upon contact. Then interactions between new neighbouring kinks are switched on. In actual simulations the neighbouring kinks and antikinks with a separation of less than $0.1w$ are assumed to have annihilated. Therefore the total number of kinks, $N(t)$, decreases with increasing time t. In addition, we impose a periodic boundary condition with a system size $L \equiv N(0)/n_0$, where n_0 is the initial number density of kinks. With $N(0)$ fixed, the changeable parameters, having physical meaning are: the initial density of kinks n_0, the initial number difference between kinks and antikinks $\Delta N \equiv N_K(0) - N_{AK}(0)$, and the velocity v^0. The initial spatial distributions of kinks, $\{y_i(0)\}$, are given by sets of random numbers. Hereafter we set $\Delta N = 1$ and $v^0/V = 0.01$. Note that the case with $\Delta N = 1$ is thought to correspond to the work by Ivlev and Kopnin [2]. In figure 1 we show an example of our numerical result for kink positions as a function of time, t, for $N(0) = 51$ and $n_0 = 0.5$ with a time step $\Delta t = 0.1$ s. Many kinks can be seen to annihilate in the early stage.

In the following simulations, to study the time-dependent behaviour of the system in a little more detail, we set $N(0) = 10^4$ and the results below are obtained by averaging over 50 independent simulation runs. In figure 2 we show the time evolution of the normalized electric field $A(t)$, defined in (20), for $n_0 = 0.5$. The variation of $A(t)$ in the late stage seems to fit the $1/\log_{10} t$ behaviour. To check the validity of the above result, we have examined simulations by varying n_0 and Δt. From the numerical results we can conclude that $A(t)$ shows the inversely logarithmic dependence on time t after a transient period $t_0 \simeq \exp(1/n_0)$ as

$$A(t) = (a/\log_{10} t) + b \qquad \text{for } t > t_0 \tag{22}$$

with $a \simeq 1.5$ and $b \simeq 1.0$. The transient period, t_0, can be interpreted as the time of a collision between a kink and an antikink at $t = 0$ [5]. As a result, we find numerically that for $v^0/V = 0.01$ and $\Delta N = 1$ the induced electric field, E_x, decays logarithmically in time and asymptotically approaches the value given by Ivlev and Kopnin. We have checked that the similar time dependence of E_x could be obtained so long as $v^0/V < 0.05$.

In summary, we have numerically studied the dynamics of magnetic flux kinks in layered HTSCs. Simulating the equation of motion for kink positions from the initially random distributed flux kinks, we have found the inversely logarithmic time dependence of the induced electric field with $\Delta N = 1$ for fairly small v^0 or the transport current j. To our knowledge, such time-dependent behaviour has not yet been reported. Systematic experimental studies taking account of this aspect are therefore highly desirable. Here we have examined just a few cases, as mentioned above. Further simulations by changing the parameters, as well as the initial conditions, are therefore needed.

The author is grateful to Professor S Maekawa for a number of useful comments.

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